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GENERAL BALANCE SHEET.

Dr. LIABILITIES.

	£	s.	d.	£	s.	d.	£	s.	d.
To present Value of all Sums Assured and Past Bonus (£2,517,981).							1,276,321	0	0
„ Capital paid up . . .	£40,000	0	0						
„ Bonus Additions thereto . .	30,000	0	0						
				70,000	0	0			
„ Present Value of Life Annuities (£6,635). .				49,068	0	0			
„ Half-year's Dividend due on Shares . . .				2,226	0	0			
„ Salaries and Sundry Accounts				1,745	0	0			
							123,039	0	0
„ Surplus, viz:—							1,399,360	0	0
To the *Proprietors,* One Tenth of the Profits to be applied by way of Increase to the ordinary Dividend during the next Five Years				6,321	0	0			
To the *Assured,* to provide for Reversionary Bonus Additions to all Policies entitled to participate.				55,831	0	0			
				62,152	0	0			
„ Balance carried forward				1,066	12	3			
							63,218	12	3
							£1,462,578	12	3

Cr. ASSETS.

	£	s.	d.	£	s.	d.
By present Value of future *Net* Premiums (£61,520)				849,883	0	0
„ Assets, as per Auditors' Report, viz.:—						
New 3 per Cent. Annuities (£53,117. 18*s.* 2*d.*)	47,806	6	1			
Cash	6,820	7	10			
Balance due by Agents.	353	1	7			
Houses and Furniture—London, Edinburgh, and Glasgow	20,284	13	4			
Mortgages and other Securities . . .	526,004	3	5			
				601,268	12	3
„ Premiums due and Interest accrued on Investments . . .				11,427	0	0
				£1,462,578	12	3

25*th December,* 1865.

NOTICES OF NEW BOOKS.

Manual of Algebra. By the Rev. JOSEPH A. GALBRAITH, M.A., Fellow of Trinity College, and Erasmus Smith's Professor of Natural and Experimental Philosophy in the University of Dublin.

We notice this elementary work in consequence of the author having so far departed from previous usage as to introduce some account of the Theory of Life Annuities and Assurance. The reasons for this course are thus stated in the preface. "In consequence of the magnitude and great importance of the subject, and also because some of its problems involve reasonings and speculations of too refined and abstract a character for the less advanced student, the author has been induced to omit the Doctrine of Chances in its general form and to substitute one of its easiest but most useful applications—the theory of Life Annuities and Assurances so far as it depends on a single life."

As regards the general character of the book, we are glad to see in the Table of Contents several subjects mentioned, such as "Multinomial Theorem (Arbogast's method)," "Calculation of logarithms (Koralek's method)," and "Horner's method of Approximation," which are not usually given in elementary books, but which may very properly find a place in them. We purpose however to confine our remarks to that portion of the work which relates to the Theory of Life Contingencies.

The first remark which suggests itself, is, that if the author's chief object was to give an illustration of the Doctrine of Chances, he would have done better not to limit himself to questions involving one life only, but to give also the elementary theorems as to the values of annuities and assurances on two and three lives, and as to the probabilities of survivorship among them. We are also inclined to think that such details as the use of the D and N columns for the calculation of annuities, which certainly do not illustrate the Doctrine of Chances, would be with greater propriety confined to works specially devoted to the subject of Life Contingencies.

On the above points some difference of opinion will perhaps exist. On another point, however, we believe that there is no room for difference of opinion. We have no doubt that all our readers will agree in our opinion that Mr. Galbraith has acted very unwisely in including in his treatise, and attempting to discuss within a few lines, a number of important questions respecting which practical actuaries are by no means agreed. Thus he has in the course of five pages disposed of the following questions.

1. Management of Life Assurance Companies.
2. The Bonus System.
3. "Valuation of Profits."
4. Advantages and Disadvantages of Bonus.
5. Division of Profits.

Each of these subjects might fairly have claimed a chapter for itself; and they are questions which can scarcely be discussed with advantage by any one who is not practically conversant with the management of Insurance Companies. Accordingly we shall see that our author in his remarks, brief as they are, has fallen into several grave errors.

Our author states that it is the "almost universal" practice of actuaries to employ the Carlisle table of mortality, and 3 per cent. as the rate of interest. The result of our observation is that the Carlisle table is much less frequently used than would appear from the above statement. We believe that the Northampton table of mortality is still used by several actuaries, although very few, if any, are now found openly to defend it, and it has been generally abandoned in favour of tables which make a closer approach to the actual facts. Among these, the *Equitable Experience* of Griffith Davies, the *English Life*, and the *Experience of* 17 *Offices*, have each their supporters. There are also several unpublished tables still in use, although the tendency of the present time is to look upon these with increasing disfavour. Lastly, it is the practice of some actuaries to value by means of an artificial table calculated from the premiums actually charged. Again, with regard to the rate of interest, though it is certainly the most usual course to adopt 3 per cent., still there are several Offices whose liabilities are valued at 3½, or even 4, per cent. On a general review of the question, we are inclined to say that the Carlisle 3 per cent. data, although probably employed more frequently than any other data that could be specified, are certainly not employed in half the valuations that are made. It is a matter of regret, however, that many Offices in publishing their Bonus Reports do not state upon what data their valuations have been made.

The most important problem which an actuary has to deal with is, the periodical valuation of the liabilities of a Life Office. We notice in passing that instead of the preceding phrase, our author uses the odd one, "Valuation of Profits." He also attaches a novel meaning to the word "Profits," saying, "the Profits are applied in the first instance to defray " the expenses of management"; while it would generally be supposed that those expenses must be defrayed before any profits can be realized. The above problem being so important, it should surely, if considered at all, be treated with the greatest care, and precision. But Mr. Galbraith's treatment of the question shows a strange confusion of ideas. That we may not be thought to misrepresent him, we will quote his own words: "The only way in which the affairs of an assurance office can be " effectually investigated is to find the value of each Policy as in Ex. 7, " p. 510 $\left[\text{i.e. by the formula } 1 - \frac{1+a_{x+n}}{1+a_x} \right]$. From the total amount of " assets subtract a sum sufficient to buy up all the policies, *and also a sum* " *sufficient to provide a fund for future management;* any balance which " remains may be considered as fair profit." It will be scarcely necessary to point out that if the policies are valued as Mr. Galbraith directs, the whole of the loading of the premiums is thrown off, and it is quite unnecessary to make any further provision for future expenses of management. Nor need we remind our readers that it is quite as satisfactory to value the policies in classes, as to value them separately, and that in several respects the former method is preferable.

The author then proceeds. "The method usually followed is " equivalent to this, although different in detail. Let P be the sum of the " present values of all the premiums payable to the Office; C the present " value of all the claims that can arise against it; A the total amount of " assets; and M the present value of the expenses of future management; " then

$$\text{“ } P+A-C-M=\text{Profit.”}$$

We do not know what ground our author may have for supposing this to be "the method usually followed"; but we can assure him that, if the method is ever used, it is in very rare and exceptional instances. Nor is this method equivalent, as he states, to the one previously explained. It would be so, if P denoted the value of the *net* premiums, but P is clearly defined to be the value of the "premiums payable to the Office." This being so, the use of this method would amount to an anticipation of all future profit on existing policies. It is usual to deduct, instead of M, a sum, L, equal to the value of the loading of the premiums, which includes a provision, not only for future expenses, but also for future profits.

Scarcely more satisfactory is the treatment of the subject of "Division of Profits," which we are informed, is "one of the most unsatisfactory " questions connected with the business of life assurance." This may, or may not, be the case; but a treatise on algebra is not the place where we should have expected the information to be given. Our author states, without any limitation, that "the only just mode of distributing a surplus " is to calculate for each policy holder a cash bonus in exact proportion to " the loading on his premium over and above his share of the expense of "management, accumulated at compound interest during the intervals of "division." He is apparently not at all aware that this method can make no claim to accuracy unless the liabilities of the Office have been valued by what are sometimes called the true table of mortality, and the true rate of interest. If, on the contrary, it is thought prudent to make the valuation by a 3 per cent. table, while the Office has really been realizing interest at the rate of, say, 4¼ per cent. on all its assets, then the above method would give results very far from equitable. We cannot on the present occasion dwell further on this interesting topic; but hope to recur to it at some future time.

We have next to notice our author's comparison of the "Advantages and Disadvantages of Bonus," which appears to us even more out of character with the nature of the book than anything we have as yet noticed. In making this comparison Mr. Galbraith supposes two persons to insure at the age of 25—the one paying a premium of £2. 6*s*. 6*d*. per £100 for an insurance with profits, and the other insuring without profits and paying £1. 16*s*. 5½*d*. per £100. His conclusion is, that it is far more advantageous to insure without profits than with profits. This conclusion may readily be admitted when the difference between the premiums is as great as is here supposed—amounting to no less than 21½ per cent. of the participating premium; but we cannot for an instant admit that this is a fair statement of the case. In fact no Office, (except under very unusual circumstances,) makes so large a difference as to load its without profit premium 7 per cent. and its with profit premium 36½ per cent.! We trust that in any future edition of the work, all reference to these disputed subjects will be carefully avoided.

With reference to the other parts of the subject, to which the preceding objections do not apply, we regret to say that we notice so many inaccuracies of various kinds that we could not at present recommend the work as a text book for students. We will not dwell upon errors such as theoretical students of a subject are sure to fall into—such, for instance, as the statement that money is never lent at compound interest. But we think that we may reasonably expect a theoretical writer, who is explaining the meaning of the "expectation of life," to be more careful than to say that, taking 1,000 persons of 32 years of age, "it is morally certain that if "their ages at death be added together, the sum will be 33,000, or there-"abouts"—instead of 65,000. Again, can anything be more vague and unsatisfactory—or indeed, more untrue—than the statement that "within "certain limits each living person possesses a tenure of life longer or shorter "according to his age"? We will not dwell on minor inaccuracies, such as speaking of an "annuity of £1 a year,"—of the "annuitant" living to receive the payment of an annuity instead of the "nominee" living—of the "number who live and die throughout the year"; but we think it is not tolerable to speak of a "reversion of £1" when meaning a "reversionary annuity of £1." Nor do we think that standard writers speak of "the first and last annuity" when they wish to denote "the first and the last payments of the annuity." If we seem to be severe in pointing out these blemishes, we would remind our readers that one of the chief benefits of a mathematical course is the precision of thought it requires of the student; and that accuracy of language is the unfailing index, if not the invariable accompaniment, of precision of thought.

The only remaining point we have to notice is that Mr. Galbraith adopts the innovation of defining N_x as equal to $D_x + D_{x+1} + \ldots.$ and yet speaks of this as "Davies's notation." This is introducing a fresh element of confusion into a question already too much involved. According to Davies's notation and formulæ

$$N_x = D_{x+1} + D_{x+2} + \ldots.$$

and the use of Davies's symbol, N_x, to denote a different quantity, is a practice much to be deprecated, as very likely to lead to great confusion and error. All that is necessary to prevent mistakes is the use of a new symbol instead of Davies's; and we would suggest $\mathbb{N}_x$ for the purpose; so that we should have

$$N_x = D_{x+1} + D_{x+2} + \ldots.$$

$$\mathbb{N}_x = D_x + D_{x+1} + D_{x+2} + \ldots.$$

and $$\mathbb{N}_x = N_x + D_x.$$

Then, a_x denoting as usual an annuity on a life, x, if $\mathbf{a}_x$ denote an annuity-due, or an annuity payable in advance, on the same life, we shall have

$$a_x = \frac{N_x}{D_x}, \quad \mathbf{a}_x = 1 + a_x = \frac{\mathbb{N}_x}{D_x}.$$

CORRESPONDENCE.

AN ASSURANCE FALLACY.

To the Editor of the Assurance Magazine.

SIR,—The following problem presents several points of interest.

An assurance of A pounds is to be effected on (x), at an annual premium (ϖ), subject to the condition that interest on the premiums paid up to and including the year of death is to be allowed by the Office, at the rate involved in the tables employed, which rate it is assumed is that realized by the Office. Required ϖ.

Attempt a solution thus:—Since all the interest realized is to be handed over to (x) or his representatives, the Office has obviously nothing but the bare premiums out of which to pay the sum assured. It is, therefore, as regards the Office, the same thing as if no interest were made; and we consequently need take account only of the average number of premiums that will be received from each policyholder. This number being $1 + e'_x$ (where e'_x is the *curtate* mean duration of lives aged x), we have

$$\varpi(1 + e'_x) = A;$$

whence $$\varpi = \frac{A}{1 + e'_x} \quad \ldots\ldots\ldots \quad (1).$$

This is a very singular result. It is independent of the rate of interest; and yet it is obvious that the higher the rate realized by the Office the